A Brief Review of Supercavitating Hydrofoils

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This paper represents a critical review on hydrodynamic aspects. Some useful mathematical techniques and various cavity models are briefly summarized. Emphasis has been made on the cause of serious discrepancies between hydrodynamic theory and experiments for supercavitating hydrofoils. Some experimental data for force measurements and flutter tests are discussed. It has been felt that some of the measurements for flutter phenomena are not reliable, and the classical lifting-surface theory should be used to obtain a more accurate flutter prediction for an actual flutter model. A few recommendations for prospective future research are also given.

Nomenclature

A = constant
a_i = acceleration vector
a_o = sound velocity in undisturbed region
b = semichord, or nondimensional semichord
b_o = semichord at the root
c = chord length
D/Dt = $(\partial/\partial t) + u(\partial/\partial x) + v(\partial/\partial y) + w(\partial/\partial z)$
$e^{i\omega t}$ = exponential function
F = complex potential
i = $(-1)^{1/2}$
k = reduced frequency
K = kernel function
M = Mach number
p = pressure
Δp = pressure difference across the vortex sheet
R = $[(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(Z - \zeta)^2]^{1/2}$
R' = $[\lambda^2 + \beta^2(y - \eta)^2 + \beta^2 Z^2]^{1/2}$
s = spanwise distance
t = time
U = uniform velocity along x-direction
u = velocity component along x-direction
v = velocity component along y-direction
w = velocity component along z-direction
W_a = up-wash velocity of the foil
$\bar{W}_a$ = amplitude of up-wash
x, y, z = Cartesian coordinates
Z = complex number
$\bar{Z}$ = complex conjugate
α' = ordinate magnitude
β = $[1 - M^2]^{1/2}$
γ = circulation
∇^2 = $(\partial^2/\partial x^2) + (\partial^2/\partial y^2) + (\partial^2/\partial y^2)$
ζ = location parallel to z-axis
η = location parallel to y-axis
λ = parameter
ξ = location parallel to x-axis
ρ = density of the fluid
σ = cavitation number
ϕ = velocity potential
ψ = acceleration potential
ω = frequency of oscillation

Subscripts

a = on the foil
c = cavitation
d = doublet
i = tensorial indices
l = leading edge
o = fundamental solution, or undisturbed region
s = sink
t = trailing edge

Superscript

$(\ )^*$ = transformed coordinate

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I. Introduction

A CRITICAL review of hydrodynamic theories and experiments in subcavitating hydrofoils has been discussed by Chu.[1] Due to the increasing speed of hydrofoil craft, special attention has been drawn to the active research in the field of unsteady, supercavitating hydrofoils. As is well known, the study of supercavitating hydrofoils is much more difficult than that of subcavitating hydrofoils. In particular, the investigation of unsteady, nonlinear, three-dimensional, supercavitating hydrofoils seems to be the most difficult problem in the realm of unsteady hydrodynamics, at least at the present time. The difficulty exists in the mathematical complexity and the experimental ingenuity required. To date, three-dimensional, unsteady, nonlinear theory of supercavitating hydrofoils has not been well established, and little experimental work has been done. On the other hand, steady two-dimensional theories of supercavitating hydrofoils are reasonably well established, and much experimental work has been successfully performed. It is therefore desirable to review briefly both the theories and experiments for the supercavitating hydrofoil. It is also hoped that this review will provide for a better understanding of the problem and some prospective recommendations for future research are made. In addition, the discrepancy between theory and experiment will be given a reasonable explanation. However a perfect physical explanation and a rigorous mathematical proof still remain to be seen.

For convenience of this discussion, we shall summarize some useful mathematical techniques and cavity models. Of course, the following classification of mathematical techniques and cavity models is selective rather than inclusive. Some mathematical techniques usually applied to the solution of hydrodynamic problems are 1) method of conformal mapping, 2) method of acceleration potential, 3) small perturbation theory, and 4) integral equation approach. It is advisable to give a more comprehensive description for each of these methods. In particular, the integral equation approach upon which the flutter estimation of finite span theory relies is extensively discussed.

1. Method of Conformal Mapping

For a two-dimensional, incompressible, irrotational flow, there exists a complex potential F, such that the Laplace equation through conformal mapping is satified, since

$$\nabla^2 F = 4(\partial^2 F/\partial Z \partial \bar{Z}) = 0 \quad (1)$$

where Z is the complex variable and $\bar{Z}$ is its conjugate. It should be borne in mind that the method of conformal mapping is not directly applicable to three-dimensional problems.

2. Method of Acceleration Potential

For an inviscid, compressible fluid and without body force, Euler's equations of motion can be written as

$$DU_i/Dt = -(1/\rho)(\partial p/\partial x_i) \qquad (i = 1,2,3) \quad (2)$$

This indicates that the acceleration vector of a fluid particle,

$$a_i = DU_i/Dt \qquad (i = 1,2,3)$$

is a gradient of a scalar quantity. Therefore, a_i can be derived from a scalar function $\psi(x_1,x_2,x_3,t)$[2], such that

$$a_i = \partial\psi/\partial x_i \qquad (i = 1,2,3) \quad (3)$$

The function ψ is said to be an acceleration potential of the flowfield.

3. Small Perturbation Theory

Small perturbation theory may be established by assuming a solution to the exact problem in the form of a power series in a perturbation parameter. It was pointed out by Tulin[3] that "the perturbation theory applied to hydrofoils differs from its normal aerodynamic counter-part in that it involves two parameters which control the perturbation rather than only one. These are the ordinate magnitude $\alpha' = t/c$ and the cavitation number $\sigma = (p - p_c)/\frac{1}{2}\rho U_c^2$."

Small perturbation theory has been applied to the solution of steady, two-dimensional, supercavitating flow by Tulin.[3] It was also utilized to solve a problem of nonsteady, two-dimensional, supercavitating flow by Wang and Wu.[4] In principle, small perturbation theory is valid also for unsteady, three-dimensional, supercavitating flow; nevertheless, the calculation may be very laborious.

4. Integral Equation Approach

In a pattern similar to Ref. 5, we may state that for two-dimensional flow over a thin airfoil at small angle of attack, the flowfield can be described by a bound vortex sheet on the foil and a wake vortex sheet behind the foil, both at $y = 0$. The resultant vorticity on the foil may be written

$$\tilde{\Gamma} = \int_{-b=1}^{b=1} \Gamma_a(x,t)dx = -\int_{b=1}^{\infty} \bar{\Gamma}_w\, e^{i\omega[t-(x-b/U)]}\, dx = \frac{ib}{k}\,\bar{\Gamma}_w\, e^{i\omega t} \quad (4)$$

where Γ_a is the vorticity on the foil, whereas Γ_w is the vorticity in the wake, per unit length, respectively. $k = \omega b/U$ is the reduced frequency. The down-wash on the airfoil from both vortex sheets is represented by an integral equation ($b = 1$)

$$W(x) = -\frac{1}{2\pi}\int_{-1}^{+1} \frac{\Gamma_a d\xi}{x-\xi} + \frac{ik\tilde{\Gamma}}{2\pi}\int_1^{\infty} \frac{e^{-ik\xi}}{x-\xi}\, d\xi \quad (5)$$

This integral equation can be solved by the inversion of the integral and the application of Hankel functions.[6] For three-dimensional flow, the integral equation approach may be summarized as follows.

a) Reissner's lifting-strip theory

After a pretty lengthy calculation, Reissner[7,8] obtained an integral equation that governs the up-wash velocity for the foil in the following form:

$$W_a = -\frac{1}{2\pi}\int_{x_l}^{x_t} \frac{\Gamma(\xi,y)}{x-\xi}\, d\xi - \frac{e^{-i(\omega\xi/U)}}{4\pi}\int_{-s}^{+s} \frac{d\Omega}{d\eta} \times K\left[\frac{\omega}{U}(y-\eta)\right] d\eta + \frac{i\omega b_0}{2\pi U}\,\Omega(y)\int_{x_l(y)}^{+\infty} \frac{e^{-i(\omega\xi/U)}}{x-\xi}\, d\xi = \bar{W}_a\, e^{i\omega t} \quad (6)$$

where K is the kernel function defined by

$$K(y') = \frac{\omega b_0}{U}\frac{1}{y'} - \frac{i\omega b_0}{Uy'}\int_0^{\infty} e^{-i\lambda} \times \left(1 - \frac{[\lambda^2 + y'^2 - y']^{1/2}}{\lambda}\right) d\lambda$$

Ω is the circulation function defined by

$$\Omega(y) = (1/b_0)\Gamma(y)e^{i(\omega/U)x_t}$$

and x_l, x_t are the location of the leading edge and trailing edge, respectively; b_0 is the semichord at the root.

b) Lifting-surface theory

In the case where small disturbances propagate in a three-dimensional, unsteady, inviscid, compressible fluid medium, the equation of motion after transforming and neglecting higher order terms may be written as[5,9]

$$\frac{\partial^2\bar{\psi}}{\partial x^{*2}} + \frac{\partial^2\bar{\psi}}{\partial y^{*2}} + \frac{\partial^2\bar{\psi}}{\partial z^{*2}} + \frac{\omega^2}{a_0^2\beta^4}\,\bar{\psi} = 0 \quad (7)$$

where a_0 is the sound velocity in the undisturbed region, $\beta^2 = 1 - M^2$, ω is the frequency, and M is the Mach number. The fundamental solution of Eq. (7) is

$$\bar{\psi}_0 = (A/R)e^{-i(\omega/a_0\beta^2)R}$$

where A is a constant and R is defined to be

$$R = [(x-\xi)^2 + \beta^2(y-\eta)^2 + \beta^2(z-\zeta)^2]^{1/2} \quad (8)$$

As mentioned previously in the Introduction, we may introduce an acceleration potential ψ such that

$$\psi = (\partial\phi/\partial t) + U(\partial\phi/\partial x) \doteq -p/\rho \quad (9)$$

Then the acceleration potential for a harmonically oscillating sink and doublet of unit strength are, respectively,[10]

$$\psi_s = \frac{1}{4\pi R}\exp i\omega\left[t + \frac{M}{a_0\beta^2}(x-\xi) - \frac{R}{a_0\beta^2}\right] \quad (10)$$

$$\psi_d = \frac{1}{4\pi}\frac{\partial}{\partial z}\left\{\frac{1}{R}\exp i\omega\left[t + \frac{M}{a_0\beta^2}(x-\xi) - \frac{R}{a_0\beta^2}\right]\right\} \quad (11)$$

Integration of Eq. (9) gives

$$\bar{\phi} = \frac{1}{U}\, e^{-i\omega x/U}\int_{-\infty}^{x} \bar{\psi} e^{i\omega\lambda/U}\, d\lambda \quad (12)$$

where ϕ is the velocity potential. Thus the velocity potential due to a doublet on the foil is

$$\phi_d = \frac{1}{4\pi}\frac{\partial}{\partial z}\left\{e^{i\omega(x-\xi)/U}\,\frac{1}{U}\int_{-\infty}^{x-\xi} \frac{1}{R'} \times \exp i\omega\left[t + \frac{\lambda}{U} + \frac{M\lambda}{a_0\beta^2} - \frac{R'}{a_0\beta^2}\right] d\lambda\right\} \quad (13)$$

where $R' = [\lambda^2 + \beta^2(y-\eta)^2 + \beta^2 z^2]^{1/2}$. The down-wash velocity contributed by a doublet of unit strength at (ξ,η) is

$$-\frac{\partial\phi_d}{\partial z} = \frac{1}{4\pi}\frac{\partial^2}{\partial z^2}\left\{e^{-i\omega(x-\xi)/U}\int_{-\infty}^{x-\xi} \exp i\frac{\omega}{U\beta^2} \times (\lambda - MR')\,\frac{1}{R}\, d\lambda\right\} \quad (14)$$

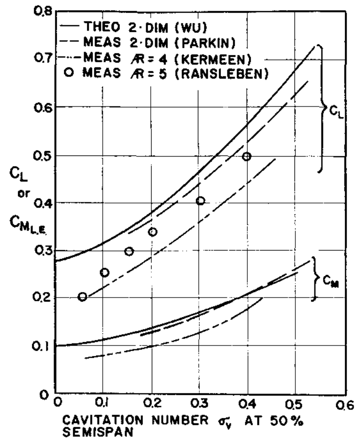

Fig. 1 Steady-state coefficients.

Since the equation is linear, we may apply the principle of superposition. If we superpose all doublets on the foil, we obtain the amplitude of the total down-wash velocity as follows:

$$\left(\frac{\partial \bar{\phi}}{\partial Z}\right)_{z=0} = \bar{W}(x,y) = \lim_{Z\to 0} \frac{1}{4\pi} \iint \frac{\Delta p}{\rho U} \left\{ e^{-i\omega(x-\xi)/U} \times \frac{\partial^2}{\partial z^2} \int_{-\infty}^{x+\xi} \frac{1}{R'} \exp i\left(\frac{\omega}{U\beta^2}\right)(\lambda - MR')\, d\lambda \right\} d\xi d\eta \quad (15)$$

where $\Delta p = p_- - p_+$ is the lift per unit area. On the foil, the up-wash

$$\bar{W}(x,y) = \dot{Z}_a = [i\omega + U(\partial/\partial x)]Z_a \quad (16)$$

Equation (15) is an integral equation which governs the lift distribution and which can be solved by the method of collocation.

In order to apply the integral equation to a supercavitating hydrofoil, we must pay special attention that the boundary conditions should be appropriately modified. In addition, free-surface and gravitational effects should also be taken into account because both Eqs. (5) and (15) are originally derived for an airfoil. The former is valid for a two-dimensional, incompressible fluid, whereas the latter is for a three-dimensional, compressible fluid.

Among the four preceding methods, the method of conformal mapping is restricted to two-dimensional problems; others may be applied to both two-dimensional and three-dimensional flow fields. Each of them has its advantages and disadvantages. A detailed discussion is beyond the scope of this paper.

Some cavity models are summarized as follows: 1) Kirchhoff-Helmhotz model (infinite), 2) modified Riabouchinsky model (finite), 3) re-entrant jet—Efros-Kreisel-Gilbarg model (finite), 4) Roshko's dissipation model (finite), 5) Wu-Fabula model (finite), and 6) Tulin's single spiral vortex and double spiral vortex model (finite). A cavity may be of infinite length. This is the case when the cavity pressure is equal to the ambient pressure. For example, the Kirchhoff-Helmhotz model is of infinite length. On the other hand, it may be of finite length if the cavity pressure is less than the ambient pressure. Models 1, 2, 3, 4, and 5 are of finite length. Strictly speaking, none of the previously mentioned models predicts accurately the flow in the region of cavity collapse, which is typically highly turbulent. However, it seems to the writer that Roshko's dissipation model and Tulin's single spiral vortex and double spiral vortex model are more physical and plausible than the others and are therefore preferred on scientific and engineering grounds.

In the following sections we shall discuss two-dimensional theories first, and then we shall review three-dimensional theories. In each case, we shall take up the calculation of forces and moments and the estimation of flutter as well.

II. Two-Dimensional Case

1. Forces and Moments Calculation

Two-dimensional theories for supercavitating hydrofoils have been applied to calculate the lift, drag, and moments by various authors. For example, Larock and Street[11] solved the problem of a steady, two-dimensional, supercavitating hydrofoil with consideration of the free surface. They used Tulin's single spiral vortex model and applied the method of conformal mapping to obtain the solution. It was Tulin[3] who used his own vortex model and applied the small perturbation theory to treat steady supercavitating flows. Tulin established both first-order theory and second-order theory; the former is identical to the usual linearized theory, the latter leads in the special cases of the inclined flat plate and symmetric wedge in supercavitating flow to results for lift and drag in simple terms of the first-order results. Brown[(30)] derived the "improved first-order supercavitating theory" for steady, two-dimension case. For unsteady, two-dimensional, supercavitating hydrofoils, Wu[12] used Roshko's dissipation model and small perturbation theory to treat an unsteady supercavitating flow. It should be pointed out that Wu's small perturbation theory is different from Tulin's in that Wu superimposed the small unsteady perturbations to a well-established steady flow whereas Tulin established a small perturbation theory for a basic steady flow. Martin[13] solved a problem of a fully submerged, supercavitating hydrofoil at zero cavitation number (cavity model of infinite length) by means of both the integral equation method and the method of conformal mapping. He obtained lift and moment coefficients. Parkin[14] used the method of conformal mapping and the complex acceleration potential to solve a problem of fully submerged, supercavitating hydrofoil in nonsteady motion. Parkin assumed a finite cavity. It was Hsu[15] who used a method similar to Parkin's to solve the problem of a two-dimensional, thin, supercavitating hydrofoil in nonuniform motion, but he used a cavity of infinite length. He has taken the free-surface effect into account. Concerning experimental investigations, Parkin and Kermeen[16] obtained some force measurements for a steady, two-dimensional, cavitating hydrofoil employing water-tunnel techniques. Lang and Daybell[17] also obtained some force measurements for a steady two-dimensional case. According to the previous results as shown in Fig. 1, the theoretical prediction has, in general, shown reasonable agreement with the experimental data for the two-dimensional steady flow of flat-plate supercavitating hydrofoils, as far as forces and moments are concerned. Unfortunately, for the unsteady, two-dimensional case, there are not enough experimental data available to be compared with theories. Song[18] did experiments for a supercavitating flat plate with an oscillating flap at zero cavitation number. He measured lift, drag, moment, and surface wave speed. He also solved the same problem analytically by means of a first-order perturbation theory, and the results were compared with experimental data. He concluded that reasonable agreement between theory, and experiments has been obtained. However, it seems to the writer that without further reliable experimental data for comparison, it would be too early to draw any general conclusion concerning the determination of forces and moments for unsteady, two-dimensional, supercavitating hydrofoil.

2. Flutter Estimation

Although considerable theoretical and experimental effort has been expended in an attempt to obtain a better understanding of hydroelastic stability—namely, flutter and divergence—no satisfactory agreement between theory and experiment has yet been obtained. Since divergence can be considered as a limit case of flutter, we shall discuss flutter only. Kaplan and Henry[19] used a two-dimensional theory to define the unsteady hydrodynamic force and moment acting on the oscillating foil. Their theory predicts that dynamic

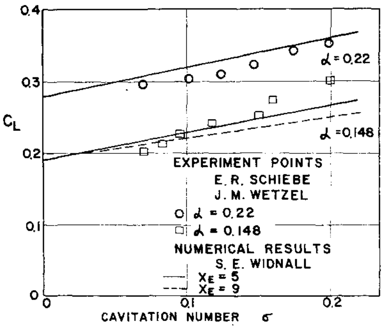

Fig. 2 Comparison between numerical results and experiment for an aspect-ratio-6 foil lift coefficient vs cavitation number σ.

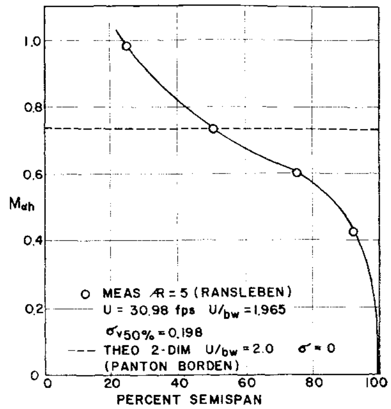

Fig. 4 Moment coefficient about $\frac{2}{3}$ chord due to rolling; $U/b\omega$ = 2.00, $\sigma_{v50\%}$ = 0.20.

instability (bending-torsion flutter) is possible at the density ratios typical of supercavitating operation. This contradicts the results for fully-wetted flow, where flutter does not possibly occur at the structural-to-fluid density ratio typical of hydrodynamic operation. They also found that for supercavitating hydrofoils, divergence and flutter are of equal importance, which again is different from the results in fully-wetted flow where divergence was shown to be more important in a practical problem. With regard to the experimental aspect, Cieslowski and Pattison[20] attempted to obtain flutter for a two-dimensional, two-degree-of-freedom, supercavitating, semiwedge section. Because of the limitation imposed by the water tunnel, they failed to observe any flutter phenomena. Only Peller and Figueroa[21] reported that they observed flutter phenomena. Their experimental setup is such that supercavitating hydrofoils were fitted with various rudders and spoils and had been flutter tested. According to Abramson,[9] their observed "flutter" conditions were doubtful, because, as the subcritical oscillations observed at the "flutter" speed disappeared rather quickly, they did not compare their experimental results with any theory; furthermore, they failed to draw any qualitative conclusion concerning supercavitating flutter. Theory by Kaplan and Henry[19] does predict the flutter of a two-dimensional, unsteady, supercavitating hydrofoil; while no experimentalist has observed flutter phenomena, this is a serious discrepancy. The reason for this is that the flow regions of unsteady, supercavitating hydrofoil are so complicated, and the experimental techniques are so difficult and ingenious that no reliable data from experiments are available at the present time.

III. Three-Dimensional Case

1. Forces and Moments Calculation

Little has been done for unsteady, nonlinear, three-dimensional, supercavitating hydrofoil. However, for linearized theory of supercavitating hydrofoils, Widnall[22] tried the analytical study. She has derived the linearized three-dimensional lifting-surface theory for a supercavitating hydrofoil with finite span in steady or oscillatory motion through an infinite fluid. She also takes the free-surface effect into account at infinite Fround number. In mathematical language, she solved the resulting coupled-integral equations on a high-speed digital computer using a numerical method of assumed modes similar to that used for fully-wetted surfaces. She compared her numerical results for lift and moments for both steady and oscillatory foils with other theories and experiments. Widnall only quoted steady, three-dimensional experiments that were carried out by Schiebe and Wetzel[23], Kermeen,[24] and Johnson[25] to compare with her theoretical prediction, as shown in Fig. 2. She did not have any nonsteady, three-dimensional experimental data to compare with at that time. According to Widnall, her numerical solution gives an efficient and accurate prediction of loads on a supercavitating foil. Other investigators have reserved their opinion pertaining to the comparison of her theory with experiments. As a matter of fact, Ransleben[26] has measured the spanwise oscillatory lift and moment distribution on a finite span, aspect-ratio-5, rectangular planform, supercavitating hydrofoil. A comparison of Ransleben's measured coefficients with Patton and Borden's[27] two-dimensional theory shows a remarkable discrepancy, as given in Figs. 3–6, respectively. This may be primarily due to the three-dimensional effect.

2. Flutter Estimation

a) Lift-strip theory

Reissner-Stevens[4] theory was derived for air-load distribution and thus it has been quite successful for the flutter prediction of an airfoil. Although a supercavitating hydrofoil is entirely different in nature from that of airfoil, it is believed that after suitable modification of the boundary conditions and with consideration of gravitational and surface effects the Reissner-Stevens theory can be possibly applied to a supercavitating hydrofoil. How good the results will be still remains to be seen.

b) Lifting-surface theory

Lifting-surface theory is based on a mathematical technique that requires solving an integral equation with a kernel function. The integral equation approach has been exten-

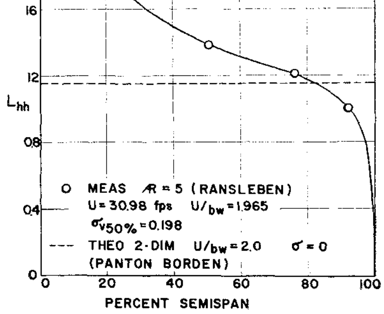

Fig. 3 Lift coefficient due to rolling; $U/b\omega$ = 2.00, σ_v = 0.20.

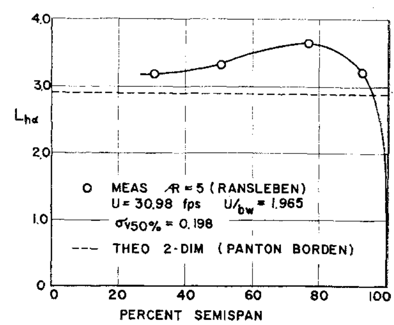

Fig. 5 Lift coefficient due to pitching about $\frac{2}{3}$ chord; $U/b\omega$ = 2.00, $\sigma_{v50\%}$ = 0.20.

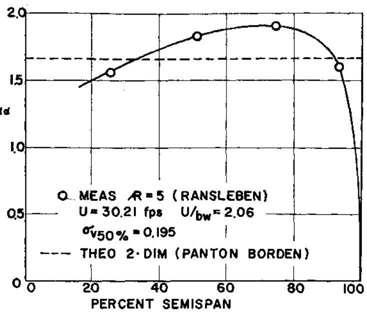

Fig. 6 Moment coefficient about $\frac{2}{3}$ chord due to pitching about $\frac{2}{3}$ chord; $U/b\omega$ = 2.00, $\sigma_{v50\%}$ = 0.20.

sively discussed in the introduction. Chu[28] has investigated the three-dimensional effect of flutter in a real fluid. Using Reissner's approximation[8] for a large-aspect-ratio rectangular wing, he obtained expressions for the down-wash, the lift, and moment per unit span, but no numerical results were given. However, no flutter predictions based on loadings from the lifting-surface theory have as yet been conducted.

Kaplan and Lehman[29] performed experimental studies of hydroelastic instabilities of supercavitating hydrofoils. They attempted to determine the flutter and divergence of cavitated finite span hydrofoils. Neither flutter nor divergence was observed in their tests.

IV. Conclusions and Discussions

Concerning forces and moments, theory and experiments for the steady two-dimensional case have shown reasonably good agreement, particularly for a two-dimensional flat-plate foil. For the steady, three-dimensional case, Widnall's linearized theory has been compared with experiments in Refs. 23–25. She claimed that there is satisfactory agreement between her linearized theory and experiments. For the unsteady, two-dimensional case, Song[18] performed experiments, and he also compared his experimental data with his theoretical results obtained by first-order perturbation theory. He claimed good agreement between theory and experiments. For the unsteady, three-dimensional case, the only established theory is the linearized theory of Widnall,[22] and, up to the present time in this country, the only available experimental data are those of Ransleben. No comparison was made between Widnall's theory and Ransleben's experiments. Although a few investigators[11–15] have established unsteady, two-dimensional theories, there are not enough reliable experimental data to verify any of those theories. A three-dimensional, unsteady, nonlinear theory for a supercavitating hydrofoil is still in the stage of infancy and very little experimental data are presently available. Therefore, it is impossible to draw any general conclusion for unsteady, two- and three-dimensional, supercavitating hydrofoils pertaining to the calculation of hydrodynamic forces and moments.

Concerning flutter phenomena of supercavitating hydrofoils, the only established two-dimensional theory is that of Kaplan and Henry,[19] and the only available experimental data were obtained by Peller and Figueroa.[21] It has been felt that the data of Peller and Figueroa are not reliable. Other investigators, such as Cieslowski and Pattison,[22] attempted to observe flutter for two-dimensional, supercavitating hydrofoils, whereas Kaplan and Lehman[29] tried to study the dynamic instabilities of the three-dimensional, supercavitating hydrofoil. Unfortunately, none of them observed any flutter phenomena. On the one hand, two- and three-dimensional unsteady theories of flutter prediction for supercavitating hydrofoils are not yet well established. The experimental techniques are so difficult and ingenious that no reliable flutter data are available at the present time.

Scale effects may be of great importance. It is evident that the water in a water tunnel or towing tank does not necessarily act on the small model of a supercavitating hydrofoil in a manner similar to that of the free water in the sea on the similar full-scale object, and thus a full-scale experimental test of a small supercavitating hydrofoil would be desirable. It should be noted once again that this paper emphasizes the hydrodynamic aspect of the unsteady supercavitating problem. Further information concerning flutter predictions may be obtained in Refs. 2, 5, and 9.

V. Recommendation on Future Research

1) A comparison of Widnall's unsteady, three-dimensional theory with Ransleben's experiments should be made.

2) A nonlinear, three-dimensional lifting-surface theory should be derived for a supercavitating hydrofoil in steady or oscillatory motion and compared with the linearized, three-dimensional lifting-surface theory of Widnall in Ref. 22.

3) A theoretical flutter analysis based on loadings from the lifting-surface theory should be completed and compared with the available experimental data.

4) Theoretical studies for improved two- and three-dimensional, unsteady viscous flow around an oscillating, supercavitating hydrofoil should be initiated and encouraged, in particular, when lifting-surface theory subject to the Kutta condition failed in the supercavitating case. This was also mentioned by Chu[1] for a subcavitating hydrofoil.

5) Experimental investigation for two- and three-dimensional flutter phenomena of unsteady, supercavitating foils should be particularly emphasized.

6) As pointed out in Ref. 9, "the treatment of boundary conditions for an unsteady supercavitating hydrofoil is an unsolved problem of great importance and equally of great difficulty." Therefore, future efforts should be expended to this aspect as well.

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